

AXION BREMSSTRAHLUNG

D.V. GAL'TSOV*, E.Yu. MELKUMOVA* and R. KERNER†

**Department of Physics, Moscow State University,
119899, Moscow, Russia*

*†Université Pierre et Marie Curie, 4, place Jussieu,
Paris, 75252, France*



Abstract

A new mechanism of cosmic axion production is proposed: axion bremsstrahlung from collisions of straight global strings. This effect is of the second order in the axion coupling constant, but the resulting cosmological estimate is likely to be of the same order as that corresponding to radiation from oscillating string loops. This may lead to a further restriction on the axion window.

1 Introduction

The cosmic axion remains one of the viable candidates for dark matter, though the range of masses, which remains open accounting for both collider and cosmological constraints, is rather narrow. The model contains an unknown parameter, the vacuum expectation value f , marking the energy scale of the U(1) symmetry breaking^{1,2,3}. The mass of the axion, which is acquired after the QCD phase transition, is inversely proportional to f , whose upper bound is of cosmological origin and follows from the requirement that axions produced during the cosmological evolution do not overclose the Universe. The lower bound on the axion mass lies between several units and several tens of μeV ; this corresponds to the value of f between 10^{12} and 10^{11} GeV. For a recent survey of the present theoretical and astrophysical status of the cosmic axion model see^{4,5}, earlier reviews include⁶.

The axion string network^{7,8} is formed at the temperature of the Peccei-Quinn phase transition f , and it is usually assumed that the reheating temperature is higher than f , otherwise the network would be diluted by inflation. Strings are primarily produced as long straight segments whose length is of the order of the horizon size, and they initially move with substantial friction^{9,10,11} due to the scattering of cosmic plasma particles. At some temperature $T_* < f$ scattering becomes negligible, and the string network enters the

scale-invariant regime^{12,13,14} when strings form closed loops and move almost freely with relativistic velocities^{15,16}. The standard estimates of the axion radiation from global strings are based on the assumption that the main contribution comes from the oscillating string loops^{17,18,19,20}.

Here we discuss another mechanism for axion radiation: bremsstrahlung, which has to be produced in collisions of long strings. The existence of such an effect can be demonstrated as follows. Consider two infinite straight strings inclined with respect to each other and moving in parallel planes. Due to interaction via the axion field, strings will be deformed around the point of minimal separation between them. The motion of this point is not associated with the propagation of any signal, and the corresponding velocity may be arbitrary, in particular, superluminal. In the second order of string-axion interaction the superluminally moving deformation must produce Cerenkov axion radiation. A similar mechanism was earlier suggested for gravitational radiation of local strings, but in that case the explicit calculations²¹ have led to a zero result. The vanishing of gravitational radiation, however, has a specific origin related to the absence of gravitons in $2 + 1$ gravity theory. Indeed, as was explained in²¹, two crossed superluminal strings can be “parallelized” by suitable coordinate transformations and world sheet reparameterizations, so that the collision of strings is essentially equivalent to the collision of point particles in $2 + 1$ dimensions. In the case of the global strings similar considerations lead us to the problem of the electromagnetic radiation of point charges in $2 + 1$ dimensions, which is not forbidden by dimensionality. Thus one can expect a non-vanishing axion bremsstrahlung from collisions of global strings.

2 Strings interacting via axion field. Axion radiation.

Consider a pair of relativistic strings $x^\mu = x_n^\mu(\sigma^a)$, $\mu = 0, 1, 2, 3$, $\sigma_a = (\tau, \sigma)$, $a = 0, 1$, where $n = 1, 2$ is the index labelling them. The 4-dimensional space-time is assumed to be flat and the signature is $+, - - -$ (and $(+, -)$ for the string world-sheets). Strings interact via the axion field $B_{\mu\nu}$ as described by the action⁸

$$S = - \sum_{n=1,2} \int \left(\frac{\mu_n^0}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a x_n^\mu \partial_b x_n^\nu \eta_{\mu\nu} + 2\pi f_n B_{\mu\nu} \epsilon^{ab} \partial_a x_n^\mu \partial_b x_n^\nu \right) d^2 \sigma_n + \frac{1}{6} \int H_{\mu\nu\lambda} H^{\mu\nu\lambda} d^4 x. \quad (1)$$

Here μ_n^0 are the (bare) string tension parameters, f_n are their axion couplings (n -labelling helps to control the perturbation expansions), the Levy-Civita symbol is chosen as $\epsilon^{01} = 1$, γ_{ab} is the metric on the world-sheets. The totally antisymmetric axion field strength is defined as $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$ with the Lorentz gauge $\partial_\lambda B^{\nu\lambda} = 0$. The corresponding potential two-form is the sum $B^{\mu\nu} = B_1^{\mu\nu} + B_2^{\mu\nu}$ of contributions $B_n^{\mu\nu}$ due to each string:

$$\eta^{\alpha\beta} \partial_\alpha \partial_\beta B_n^{\mu\nu} = -4\pi J_n^{\mu\nu}, \quad (2)$$

where $J_n^{\mu\nu} = \frac{f_n}{2} \int V_n^{\mu\nu} \delta^4(x - x_n(\sigma_n)) d^2 \sigma_n$ and $V_n^{\mu\nu} = \epsilon^{ab} \partial_a x_n^\mu \partial_b x_n^\nu$.

In the gauge $\gamma_{ab} = \eta_{ab}$ the renormalized strings equations of motion read :

$$\mu_n (\partial_\tau^2 - \partial_\sigma^2) x_n^\mu = 2\pi f_n H_n^{\mu\nu\lambda} V_{\nu\lambda}^{n'}, \quad n \neq n', \quad (3)$$

where the field $H_n^{\mu\nu\lambda}$ corresponds to contribution of the n' -th string (no sum over n'). The corresponding constraint equations are $\dot{x}^2 + x'^2 = 0$, $\dot{x}^\mu x'^\nu \eta_{\mu\nu} = 0$ for each string, where dots and primes denote the derivatives over τ and σ . The *self-action* terms in the equations of motion diverge both near the strings and at large distances, so two regularization parameters δ and Δ have to be introduced²³ which are absorbed by the classical renormalization of the string tension as

$$\mu = \mu_0 + 2\pi f^2 \log(\Delta/\delta). \quad (4)$$

The ultraviolet cutoff length δ is of the order of the string thickness $\delta \sim 1/f$, while the infrared cutoff distance Δ usually is chosen as the correlation length of the string network.

Assuming that such a renormalization is performed, we are left with the same equations of motion (3), now with the physical tension parameters μ_n , in which only the *mutual* interaction terms should be used in the right hand side.

Our calculation follows the approach of²¹ and it consists in constructing solutions of the string equations of motion and the axion field iteratively in terms of the string-axion field interaction constant (equal to f). We would like to mention also a similar perturbative approach to solve the problem of gravitating point masses in terms of geodesic deviations²². In the zero order approximation the strings are moving freely, so the lowest order axion fields due to both strings ${}_1B_n^{\mu\nu}$ describe their mutual interaction and do not contain the radiative part. Substituting them to (3), we obtain the first order deformations of the world-sheets. These are used to build the first order source terms ${}_1J_n^{\mu\nu}$ in the axion field equations (2) which generate the second order axion fields ${}_2B_n^{\mu\nu}$ already containing the radiative parts. The radiation power can be computed as the reaction work produced by the half sum of the retarded and advanced fields upon the source and presented in the standard form²⁴

$$\mathcal{E} = \frac{1}{\pi} \int k^0 \epsilon(k^0) |{}_1J_{\alpha\beta}(k)|^2 \delta(k^2) d^4k, \quad (5)$$

where ϵ is the sign function.

The final formula for the axion bremsstrahlung from the collision of two global strings can be obtained analytically in the case of the ultrarelativistic collision with the Lorentz factor $\gamma = (1 - v^2)^{-1/2} \gg 1$. The spectrum has an infrared divergence due to the logarithmic dependence of the string interaction potential on distance, so a cutoff length Δ has to be introduced. The final formula for the radiated energy per unit length of the target string reads:

$$\frac{d\mathcal{E}}{dl} = \frac{16\pi^5 f^6 \kappa}{3\mu^2} \phi(y), \quad y = \frac{d}{\gamma \kappa \Delta}, \quad \kappa = \gamma \cos \alpha, \quad (6)$$

here α is the strings inclination angle and

$$\phi(y) = 12\sqrt{\frac{y}{\pi}} {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -y\right) - 3\ln(4ye^C) + \frac{7}{2},$$

where the generalized hypergeometric function is introduced and C is the Euler's constant. The leading term for small y is the logarithm, while the asymptotic value for large y is constant $\phi(\infty) = 7/2$.

The velocity of the effective source (i.e. of the point of minimal separation between the strings) tends to infinity when the string become parallel. In this case, corresponding to $\kappa = \gamma$ in the above formula, the Cerenkov cone opens up to $\theta = \pi/2$, and the whole picture becomes essentially $2 + 1$ dimensional. This gives an alternative description of the effect as bremsstrahlung under collision of point electric charges in $2 + 1$ electrodynamics²⁴.

3 Cosmological estimates

Our mechanism presumably works in the temperature interval (T_*, T_1) between the onset of the scaling regime till the QCD phase transition, though an additional contribution from the damped epoch $T_0 < T < T_*$ can also be expected. The scaling density of strings is determined from numerical experiments, it can be presented as $\rho_s = \zeta \mu / t^2$ with ζ varying from 1 to 13 in different simulations. The corresponding moments of time are commonly estimated as⁸

$$t_*(\text{sec}) \sim 10^{-20} \left(\frac{f}{10^{12}\text{GeV}} \right)^{-4}, \quad t_1(\text{sec}) \sim 2 \cdot 10^{-7} \left(\frac{f}{10^{12}\text{GeV}} \right)^{1/3}. \quad (7)$$

Consider the scattering of an ensemble of randomly oriented straight strings on a selected target string in the rest frame of the latter. Since the dependence of the string bremsstrahlung on the inclination angle α is smooth, we can use for a rough estimate the result for parallel strings ($\alpha = 0$) introducing an effective fraction ν of “almost” parallel strings (roughly $1/3$). Then, if there are N strings in the normalization cube $V = L^3$, we have to integrate the

radiation energy released in the collision with the impact parameter $d = x$ over the plane perpendicular to the target string using the measure $N/L^2 \cdot 2\pi x dx$. Actually we need the radiation power per unit time, so, for an estimate, we have to divide the integrand on the impact parameter. This quantity should be multiplied by the total number of strings N to get the radiation energy released per unit time within the normalization volume. Therefore, for the axion energy density generated per unit time we obtain:

$$\frac{d\varepsilon_a}{dt} = \int_0^L \mathcal{E} \nu \frac{N}{L^2} \frac{N}{V} 2\pi x dx. \quad (8)$$

Substituting here the Eq. (5), we integrate over the impact parameter, taking into account that the string number density is related to the energy density via $N/V = \rho_s/\mu L$. Then introducing the integral $\xi(w) = \int_0^w \phi(y) dy$ and keeping only the second (leading) term in the expression (4) for the string tension μ , we obtain

$$\frac{d\varepsilon_a}{dt} \sim K f^2 \gamma^3 \xi(w), \quad w = \frac{L}{\gamma^2 \Delta}, \quad (9)$$

where $K = \frac{8}{3}\pi^4 \nu \zeta^2$. Since in the cosmological context $L \sim \Delta \sim t$, one can take $w = \gamma^{-2}$ for an estimate. The “realistic” value of γ is of the order of unity, while our formulas were obtained in the $\gamma \gg 1$ approximation. But an independent calculation shows that for $\gamma \sim 1$ the contribution from the first term is of the same order, so we can hope to get reasonable order of magnitude estimate using the above formulas for γ not very large. The full γ -dependence is given by the function $\gamma^3 \xi(\gamma^{-2})$, which for $\gamma = \sqrt{2}$ is $\gamma^3 \xi(\gamma^{-2}) \approx 8.137$, while for $\gamma = 5$, $\gamma^3 \xi(\gamma^{-2}) \approx 55.83$. Another source of uncertainty is the numerical value of the parameter ζ , which was obtained in different simulations within the region between 1 and 13.

Integrating the radiation power over time from t_* to t_1 as given by (7) one finds the energy density of axions at the moment of QCD phase transition $t_1 \gg t_*$:

$$\varepsilon_a \sim \frac{K f^2 \gamma^3 \xi(w)}{2t_*^2 \ln^2(t_*/t_0)}. \quad (10)$$

Finally, dividing this by the critical energy density at $t = t_1$ $\varepsilon_{\text{cr}} = \frac{3m_{\text{Pl}}^2}{32\pi t_1}$, $m_{\text{Pl}} = 1.22 \cdot 10^{19} \text{GeV}$, and using the relation $t_*/t_0 = (m_{\text{Pl}}/f)^2$, we obtain for $\gamma = \sqrt{2}$ the following estimate for the relative contribution of the bremsstrahlung axions

$$\Omega_a^{\text{br}} \sim 0.5 \cdot 10^{16} \left(\frac{\zeta}{13} \right)^2 \left(\frac{f}{10^{12} \text{GeV}} \right)^{32/3}. \quad (11)$$

This rough estimate shows that our new mechanism is very sensitive to the value of f and gives an upper bound on the axion window of the order of several 10^{10}GeV . We postpone more accurate cosmological estimates for a separate publication. In particular, more careful analysis is needed to accommodate the analysis to the actual astrophysical value of the string Lorentz factor.

References

1. R. Peccei and H. Quinn, *Phys. Rev. Lett* **38**, 1440 (1977); *Phys. Rev. D* **16**, 1791 (1977).
2. S. Weinberg, *Phys. Rev. Lett* **40**, 223 (1978).
3. F. Wilczek, *Phys. Rep.* **40**, 279 (1978).
4. M. Srednicki, *Axions: past, present and future*, in *Proceedings of the International Conference on Continuous advances in QCD, Minneapolis 2002*, edited by K. A. Olive, M. A. Shifman, M. B. Voloshin (Singapore, World Scientific, 2002.), p. 509; hep-th/0210172.
5. P. Sikivie, *Axions and their distribution in galactic halos*, *Invited talk at 4th International Workshop on the Identification of Dark Matter (IDM 2002)*, York, England, 2002, 10pp.; hep-ph/0211254.

6. J.E. Kim, *Phys. Rep.* **150**, 1 (1987); H.-Y. Cheng, *Phys. Rep.* **158**, 1 (1988); M.S. Turner, *Phys. Rep.* **197**, 67 (1990); G.G. Rafflet, *Phys. Rep.* **198**, 1 (1990).
7. M.B. Hindmarsh and T.W.B. Kibble, *Rept. Prog. Phys.* **58**, 477 (1995); hep-ph/9411342.
8. R. A. Battye and E. P. S. Shellard, *Recent perspectives on axion cosmology*, in *Proceedings of the International Conference on Dark Matter in Astro- and Particle Physics, Heidelberg, 1996*, edited by H. V. Klapdor-Kleingrothaus (World Scientific, 1997), p. 554; astro-ph/9706014.
9. R. L. Davis and P. Sikivie, *Phys. Lett. B* **195**, 361 (1987).
10. A. Vilenkin, *Phys. Rev. D* **43**, 1061 (1991).
11. C.J.A.P. Martins and E.P.S. Shellard, *Phys. Rev. D* **53**, 575 (1996); hep-ph/9507335.
12. C. Hagmann and P. Sikivie, *Nucl. Phys. B* **363**, 247 (1991).
13. M. Nagasawa, *Prog. Theor. Phys. D* **8**, 851 (1997); hep-ph/9712341.
14. M. Yamaguchi, M. Kawasaki, and J. Yokoyama, *Phys. Rev. Lett* **82**, 4578 (1998);
15. A. Albrecht and N. Turok, *Phys. Rev. Lett* **54**, 1868 (1985).
16. E.P.S. Shellard, *Nucl. Phys. B* **283**, 624 (1987).
17. R.L. Davis and E.P.S. Shellard, *Nucl. Phys. B* **324**, 167 (1989).
18. A. Dabholkar and M. Quashnock *Nucl. Phys. B* **333**, 815 (1990).
19. M. Yamaguchi, M. Kawasaki, and J. Yokoyama, *Phys. Rev. Lett* **82**, 4578 (1999).
20. C. Hagmann, S. Chang, and P. Sikivie, *Phys. Rev. D* **63**, 125018 (2001).
21. D.V. Gal'tsov, Yu.V. Grats, and P.S. Letelier, *Ann. of Phys.* **224**, 90 (1993).
22. R. Kerner, J.W. van Holten, and R. Colistete Jr, *Class. Quant. Grav.* **18**, 4725 (2001).
23. R.A. Battye and E.P.S. Shellard, *Phys. Rev. Lett* **75**, 4354 (1995).
24. D.V. Gal'tsov, E.Yu. Melkumova and R. Kerner, in preparation.